International Journal of Theoretical Physics, Vol. 41, No. 5, May 2002 (© 2002)

Cavity Field Spectra of the Intensity-Dependent Two-Mode Jaynes–Cummings Model

Yun-Feng Gao,[1,4] Jian Feng,[2,3] and Shu-Ren Shi[1]

Received October 4, 2001

The cavity field spectrum of a two-level atom interacting with two modes of the radiation field through intensity-dependent coupling in an ideal cavity is investigated. The results for the initial fields in pure number states, coherent states, and squeezed vacuum states are calculated. We find that the frequency of one mode is tuned by the intensity of the other mode when the two modes are both in pure number states or coherent states. A complicated multipeak structure appears when both field modes are in a superposition of number states initially.

KEY WORDS: two-mode Jaynes–Cummings model; intensity-dependent coupling; cavity field spectrum.

1. INTRODUCTION

The Jaynes–Cummings model (JCM; Jaynes and Cummings, 1963), which comprises a single two-state atom interacting with a single near-resonant quantized cavity mode of the electromagnetic field, together with its various modifications and generalizations, play a central role in quantum optics today. Numerous nonclassical effects including the vacuum field Rabi splitting (Agarwal, 1985) have been predicted. The JCM has been corroborated experimentally (Meschede *et al.*, 1985) and the vacuum field Rabi splitting observed (Thompson *et al.*, 1992). Recently, the two-photon interaction of a single two-level atom with the quantized electromagnetic field has been the subject of considerable study (Gou, 1990; Luo *et al.*, 1993; Puri and Agarwal, 1988) because of its importance in revealing the

[1] Department of Educational Technology, Liaocheng Teachers University, Liaocheng 252059, People's Republic of China.
[2] Laser Spectroscopy Laboratory, Anhui Institute of Optics and Fine Mechanics, Chinese Academy of Sciences, Hefei 230031, People's Republic of China.
[3] Institute of Optical Communication, Liaocheng Teachers University, Liaocheng 252059, People's Republic of China.
[4] To whom correspondence should be addressed at Department of Educational Technology, Liaocheng Teachers University, Liaocheng 252059, Shandong Province, People's Republic of China; e-mail: jygcx@lc-public.sd.cninfo.net.

0020-7748/02/0500-0867/0

nonclassical properties of multiphoton transitions of atoms and the experimental development of a two-photon micromaser (Netto *et al.*, 1991). Since the construction of the first single-mode two-photon laser by Gauthier *et al.* (1992) based upon dressed atom states, considerable attention has been focused on the nondegenerate two-photon JCM for realizing a two-mode two-photon laser. It is well known that in two-photon processes, intensity-dependent effects must be taken into account, so Napoli and Messina generalized the intensity-dependent JCM (Buck and Sukumar, 1981) to a two-mode cavity field with a two-photon process and studied its dynamic properties (Napoli and Messina, 1996).

On the other hand, the "physical spectrum" (Eberly and Wodkiewcz, 1977), defined by Eberly and Wodkiewicz, has been studied widely because much of the information obtained in experiments comes in the form of spectral data. The cavity field spectra of the two-photon JCM (Nasreen and Razmi, 1993), the nondegenerate two-photon JCM (Ashraf, 1994), and the two-atom two-photon JCM (Gao *et al.*, 1999) were studied in recent years. But the effect of the intensity-dependent property of the coupling coefficient was not taken into account. In this paper, therefore, we study the cavity field spectra of the intensity-dependent two-mode two-photon JCM (Napoli and Messina, 1996). The result shows that one can use the field intensity of one mode to modulate or control the frequency of the other mode quite sensitively. This influence of one mode on the other is in marked contrast to the cavity field spectrum of the two-mode two-photon JCM in which intensity-dependent effects are neglected.

2. MODEL AND CAVITY FIELD SPECTRUM

We consider a two-level atom consisting of upper level $|+\rangle$ and lower level $|-\rangle$, with intensity-dependent coupling between two modes of a radiation field through a two-photon resonant process, i.e., $\omega_0 = \omega_1 + \omega_2$ where ω_0 and ω_i ($i = 1, 2$) are the frequencies of the atomic transition and the ith mode of the radiation field, respectively. Photons are either emitted or absorbed in pairs by the atom. The Hamiltonian for such a system in the rotating-wave approximation is written as (Napoli and Messina, 1996)

$$H = \sum_{i=1}^{2} \omega_i \left(a_i^+ a_i + \frac{1}{2} \right) + \frac{1}{2}\omega_0 \sigma_z + g \left(\sqrt{a_1^+ a_1 a_2^+ a_2}\; a_1^+ a_2^+ \sigma + \sigma^+ a_1 a_2 \sqrt{a_1^+ a_1 a_2^+ a_2} \right) \tag{1}$$

where $\hbar = 1$, a_i and a_i^+ are the annihilation and creation operators for the ith mode of the field, and σ_z, σ, and σ^+ are the pseudospin operators for the atom. The constant g combined with the operator $\sqrt{a_1^+ a_1 a_2^+ a_2}$ plays the role of the intensity-dependent coupling constant between the atom and the two-mode cavity field.

Let

$$H\left|\phi_j^N\right\rangle = E_j^N\left|\phi_j^N\right\rangle \quad (j = 1, 2) \tag{2}$$

The eigenvector of Hamiltonian (1) may be represented by

$$\left|\phi_j^N\right\rangle = \sum_{m=1}^{2} C_{jm}^N\left|\Psi_m^N\right\rangle \quad (j = 1, 2) \tag{3a}$$

where

$$\begin{aligned}\left|\Psi_1^N\right\rangle &= |+, n_1, n_2\rangle \\ \left|\Psi_2^N\right\rangle &= |-, n_1 + 1, n_2 + 1\rangle\end{aligned} \tag{3b}$$

and $|n_i\rangle$ is the photon-number state of the ith mode field and N represents (n_1, n_2).

With the help of (2) and (3) the Hamiltonian (1) may be diagonalized and the result is

$$E_j^N = \omega_1(n_1 + 1) + \omega_2(n_2 + 1) + \lambda_j^N g \quad (j = 1, 2) \tag{4a}$$

with

$$\lambda_j^N = \pm(n_1 + 1)(n_2 + 1) \quad (j = 1, 2) \tag{4b}$$

and

$$C_{11} = C_{12} = C_{21} = -C_{22} = \frac{1}{\sqrt{2}} \tag{4c}$$

since C_{jm}^N is not related to n_i, we can omit its superscript N.

Following the definition of the physical spectrum by Eberly and Wodkiewicz, the cavity field spectrum may be written (Ashraf, 1994; Nasreen and Razmi, 1993)

$$\begin{aligned}S(\omega) = 2\Gamma \int_0^T dt'\, e^{-(\Gamma - i\omega)(T - t')} \int_0^T dt\, e^{-(\Gamma + i\omega)(T - t)} \\ \times \langle\Phi(0)|A^+(t')A(t)|\Phi(0)\rangle\end{aligned} \tag{5}$$

where Γ is the bandwidth of the spectrometer, T is the time at which the measurement takes place, and $|\Phi(0)\rangle$ is the initial state of the system $A(t) = a_1(t) + a_2(t)$.

Suppose that the atom is initially in its upper level and the two modes of the field in mixtures of number states, then the initial wavefunction of our system $|\Phi(0)\rangle$ can be written as

$$|\Phi(0)\rangle = \sum_{n=0}^{\infty} q_{n_1}^{(1)} q_{n_2}^{(2)} |+, n_1, n_2\rangle \tag{6}$$

where $q_{n_i}^{(i)} (i = 1, 2)$ is the ith mode number-state expansion coefficient.

We calculate the cavity field spectrum and obtain

$$S(\omega) = 2\Gamma \sum_{n_1=-1}^{\infty} \sum_{n_2=-1}^{\infty} \sum_{m=1}^{2} \left| \sum_{j=1}^{2} q_{n_1+1}^{(1)} q_{n_2}^{(2)} G_{jm}^{N_1} Z_{jm}^{N_1} + q_{n_1}^{(1)} q_{n_2+1}^{(2)} F_{jm}^{N_2} Y_{jm}^{N_2} \right|^2 \tag{7a}$$

$$G_{jm}^{N_1} = \sum_{k=1}^{2} C_{j1} C_{jk} b_k^{(1)} C_{mk} \tag{7b}$$

$$F_{jm}^{N_2} = \sum_{k=1}^{2} C_{j1} C_{jk} b_k^{(2)} C_{mk} \tag{7c}$$

$$Z_{jm}^{N_1} = \frac{e^{i(\omega-\omega_1-g\Omega_{jm}^{(1)})T} - e^{-\Gamma T}}{\Gamma + i\left(\omega - \omega_1 - g\Omega_{jm}^{(1)}\right)} \tag{7d}$$

$$Y_{jm}^{N_2} = \frac{e^{i(\omega-\omega_2-g\Omega_{jm}^{(2)})T} - e^{-\Gamma T}}{\Gamma + i\left(\omega - \omega_2 - g\Omega_{jm}^{(2)}\right)} \tag{7e}$$

where $b_k^{(1)} = \sqrt{n_1 + k}$, $b_k^{(2)} = \sqrt{n_2 + k}$, $\Omega_{jm}^{(1)} = \lambda_j^{N_1} - \lambda_m^{N}$, $\Omega_{jm}^{(2)} = \lambda_j^{N_2} - \lambda_m^{N}$ with N_1 representing $(n_1 + 1, n_2)$, and $N_2(n_1, n_2 + 1)$.

In the case of one of the two mode initial states being in a pure number state or a squeezed vacuum state, one or both of $q_{n_i+1}^{(i)}$ and $q_{n_i}^{(i)}$ will be zero. Noticing Eq. (7a), we have

$$S(\omega) = S_1(\omega) + S_2(\omega) \tag{8a}$$

where

$$S_1(\omega) = 2\Gamma \int_0^T dt'\, e^{-(\Gamma - i\omega)(T-t')} \int_0^T dt\, e^{-(\Gamma+i\omega)(T-t)} \times \langle \Phi(0)|a_1^+(t')a_1(t)|\Phi(0)\rangle \tag{8b}$$

$$S_2(\omega) = 2\Gamma \int_0^T dt'\, e^{-(\Gamma - i\omega)(T-t')} \int_0^T dt\, e^{-(\Gamma+i\omega)(T-t)} \times \langle \Phi(0)|a_2^+(t')a_2(t)|\Phi(0)\rangle. \tag{8c}$$

3. RESULTS AND DISCUSSION

3.1. Initial Fields Both Pure Number States

Let us first discuss the cavity field spectra for pure number states with different n_1 and n_2. Similar to the derivation of Eq. (7), from Eq. (8) we obtain the expressions for the position and the relative height of the peaks in the spectrum of mode I (S_1) as shown in Table I. Of course, the spectrum of Mode II (S_2) can be obtained by exchanging $n_1 \leftrightarrow n_2$ in the table.

Table I. The Positions and Relative Heights of the Peaks in the Spectrum of Mode I (S_1)

	Positions ($\omega - \omega_1$)	Relative heights
$\Omega_{1,2}$	$\pm(n_2+1)g$	$2n_1+1+2[n_1(n_1+1)]^{1/2}$
$\Omega_{3,4}$	$\pm(2n_1+1)(n_2+1)g$	$2n_1+1-2[n_1(n_1+1)]^{1/2}$

Table I shows that the heights of the peaks $\Omega_{3,4}$ are much lower than the heights of peaks $\Omega_{1,2}$, and so they can be ignored. The spectral structure for one mode is two peaks. The positions of the peaks are determined by n_2 but the heights are determined by n_1. This property provides a possibility for us to modulate the frequency of the cavity field spectrum of one mode by changing the field intensity of the other mode while keeping the former mode intensity constant. To our knowledge, such an effect has not been predicted before for the cavity field spectra of the two-mode standard JCM (Ashraf, 1994). This is also in marked contrast to the spectra of the two-atom two-mode standard JCM. In the latter, the modulation is not sensitive and only modulates the lower outer two peaks of the three peaks for each mode (Gao *et al.*, 2000).

Figure 1 shows the cavity field spectra for both fields in pure number states. The split in the spectrum of mode II is linear with n_1. The coupling constant g_0 is about several megahertz (Gauthier *et al.*, 1992; Thompson *et al.*, 1992) for the standard JCM or two-photon JCM. Although the coupling constant g for the intensity-dependent two-mode two-photon process may be smaller than g_0, the frequency modulation effect would be measurable when n_i is big enough because the frequency shift is equal to $(n_i+1)g$.

3.2. Initial Fields Both Coherent States

For a coherent field the photon number distribution

$$\left|q_{n_i}^{(i)}\right|^2 = \exp(-\bar{n}_i)\frac{\bar{n}_i^{n_i}}{n_i!} \tag{9}$$

has a maximum at $n_i = \bar{n}_1$ ($i = 1, 2$), where $\bar{n}_i$ is the ith mode average photon number. The cavity field spectra for initial coherent states are shown in Fig. 2. For equal $\bar{n}_i$ the spectra show a complex multipeak structure. If we focus on the outline of the spectrum, it can be regarded as a double-peak structure for each mode. The shift in position of the peaks with $\bar{n}_i$ is similar to the case of the number state fields with n_i. The number of peaks in one mode increases with the increase in field intensity of the other mode. If one mode is in the vacuum state (Fig. 2(b)), its peak height diminishes rapidly with increase of the other mode intensity. This indicates that the coherent mode suppresses the Rabi peak of the vacuum mode. Such a multipeak structure of the intensity-dependent two-mode JCM for initial

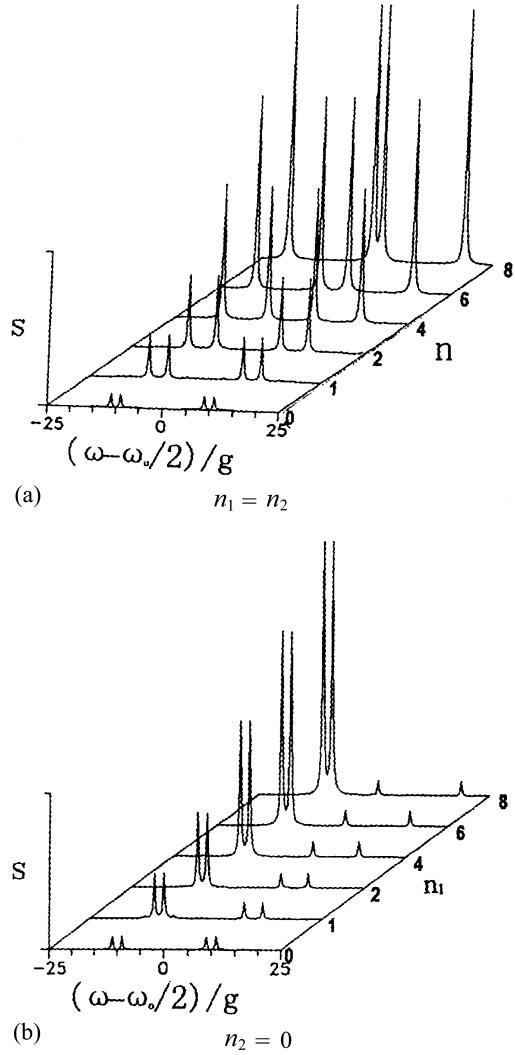

Fig. 1. Cavity field spectrum when (a) both initial fields are in a number state and (b) one of the fields is in a vacuum state and the other in a number state. Parameters: $\Gamma = 0.2g$, $\mathrm{T} = 20g^{-1}$, $\omega_2 - \omega_1 = 20g$.

coherent fields is quite different from the structure of the standard two-mode JCM, which at large $\bar{n}$ has a central double-peak structure for equal $\bar{n}_i$ or classical resonance fluorescence spectra if one mode was initially in the vacuum state (Ashraf, 1994).

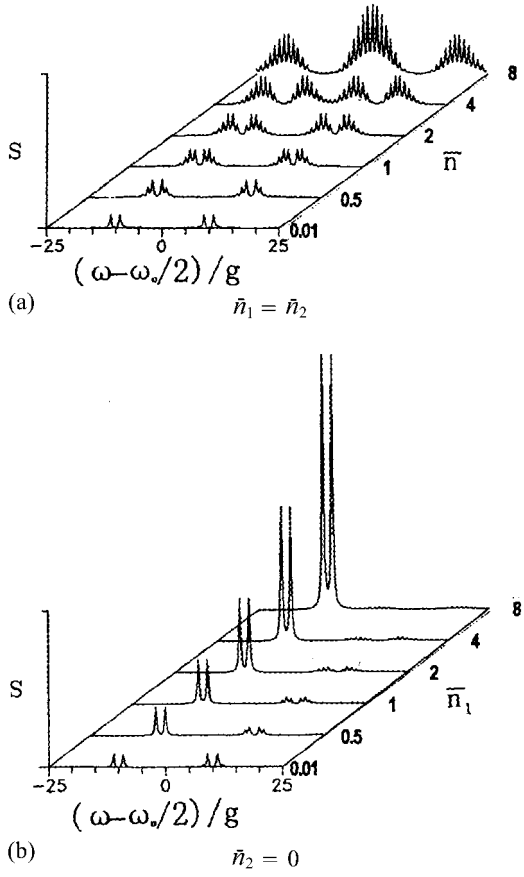

Fig. 2. Cavity field spectrum when (a) both initial fields are in a coherent state and (b) one of the fields is in a vacuum state and the other in a coherent state. Parameters: $\Gamma = 0.2g$, $\mathrm{T} = 20g^{-1}$, $\omega_2 - \omega_1 = 20g$.

3.3. Initial Fields Both Squeezed Vacuum States

For a squeezed vacuum field the photon number distribution is given by

$$\left|q_{n_i}^{(i)}\right|^2 = \begin{cases} \frac{1}{\sqrt{1+\bar{n}_i}} \left(\frac{\bar{n}_i}{1+\bar{n}_i}\right)^{n_i/2} \frac{n_i!}{2^{n_i}\left(\frac{n_i}{2}!\right)^2} & n_i = \text{even} \\ 0 & n_i = \text{odd} \end{cases} \tag{10}$$

where $\bar{n}_i$ $(i = 1, 2)$ is the average photon number in the ith mode. The situation for initially squeezed vacuum fields is shown in Fig. 3. Although the spectral structure is multipeak, the highest two peaks are always located at $\omega_i \pm g$ for both

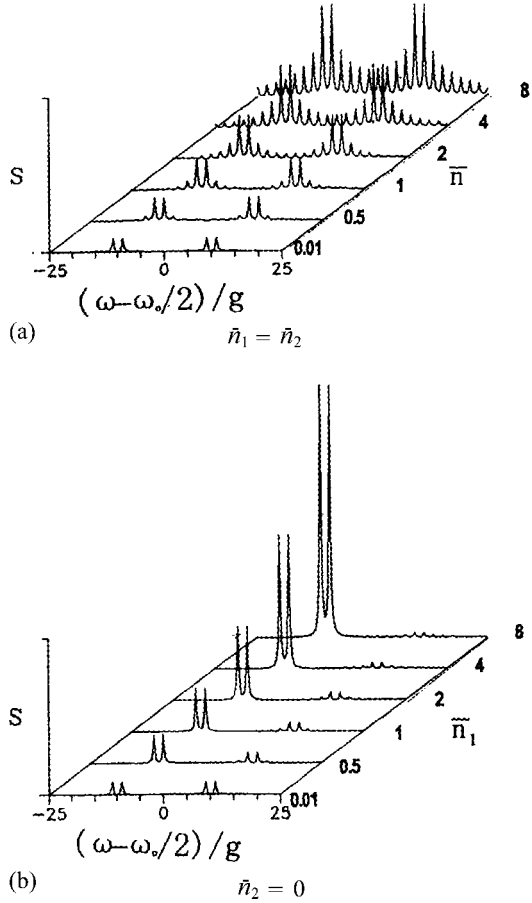

Fig. 3. Cavity field spectrum when (a) both initial fields are in a squeezed vacuum state and (b) one of the fields is in a vacuum state and the other in a squeezed vacuum state. Parameters: $\Gamma = 0.2g$, $\mathrm{T} = 20g^{-1}$, $\omega_2 - \omega_1 = 20g$.

fields in a squeezed state. Similar to the coherent state spectra, the number of peaks in one mode increases with increasing average photon number of the other mode. The suppression of the vacuum mode by the squeezed vacuum mode and growth of the Rabi peak also occurs (see Fig. 3(b)). In addition, if one field is a squeezed vacuum state while the other is not, the higher peaks of the squeezed vacuum field will generally split as the average photon number of the other field increases.

4. CONCLUSIONS

We have discussed the cavity field spectra for a two-level atom in a high-Q cavity interacting with two-mode fields via intensity-dependent coupling. When the fields are initially pure number states, two peaks of equal height for each mode are symmetrically located at $\omega_i \pm (n_j + 1)g$ $(i, j = 1, 2; i \neq j)$. The cavity field spectral frequency of one mode can be tuned by the intensity of the other mode. When the fields are in a superposition of number states, a multipeak structure appears for each mode. The number of peaks in one mode can be changed by varying the average photon number of the other mode. For coherent states, the peak position of one mode shifts with the change in initial field intensity of the other mode. However, the two highest peaks of each mode are always located at $\omega_i \pm g$ for both fields initially in a squeezed vacuum state.

ACKNOWLEDGMENT

This work was supported by the Natural Science Foundation of Shandong Province of China.

REFERENCES

Agarwal, G. S. (1985). *Journal of Optical Society of America B* **2**, 480.
Ashraf, M. M. (1994). *Physical Review A* **50**, 5116.
Buck, B. and Sukumar, C. V. (1981). *Physics Letters A* **81**, 132.
Eberly, J. H. and Wodkiewcz, K. (1977). *Journal of Optical Society of America A* **67**, 1252.
Gao, Y.-F., Feng, J., and Song, T.-Q. (1999). *Acta Physica Sinica* **48**, 1650 (in Chinese).
Gao, Y.-F., Feng, J., and Song, T.-Q. (2000). *Acta Optica Sinica* **20**, 1194 (in Chinese).
Gauthier, D. J., Wu, Q. L., Morrin, S. E., and Mossberg, T. W. (1992). *Physical Review Letters* **68**, 464.
Gou, S. C. (1990). *Journal of Modern Optics* **37**, 1469.
Jaynes, E. T. and Cummings, F. W. (1963). *Proceedings of IEEE* **51**, 89.
Luo, Z. F., Xu, Z. Z., and Xu, L. (1993). *Communications Theoretical Physics* **19**, 385.
Meschede, D., Walther, H., and Muller, G. (1985). *Physical Review Letters* **54**, 551.
Napoli, A. and Messina, A. (1996). *Journal of Modern Optics* **43**, 649.
Nasreen, T. and Razmi, M. S. K. (1993). *Journal of Optical Society of America B: Optical Physics* **10**, 1292.
Netto, P. A. M., Davidovich, L., and Raimond, J. M. (1991). *Physical Review A* **43**, 5073.
Puri, R. R. and Agarwal, G. S. (1988). *Physical Review A* **37**, 3879.
Thompson, R. J., Rempe, G., and Kimble, H. J. (1992). *Physical Review Letters* **68**, 1132.